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CYLINDRICAL TANKS

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APPROVED:



H. Norman Abramson, Director
Department of Mechanical Sciences

INTRODUCTION

The present paper is a continuation of studies of liquid sloshing in compartmented cylindrical tanks (1). Experimental data on frequencies and total force response for a 45° sector compartmented tank are presented and correlated with theoretical frequencies and with similar data for 90° sector (quarter) tanks. This data shows, as in (1), the strong dependence of liquid natural frequency on excitation amplitude and a large decrease in natural frequency with increasing equivalent Reynolds number (based on perforation hole size). The experimental equipment and procedures are similar to those employed in (1) and (2).

TANK CONFIGURATIONS

The tank configurations considered here are again cylindrical with flat bottoms, filled with liquid to a depth h , and under an axial acceleration field a . The 45° sector configuration is obtained by vertical walls extending from above the liquid free surface to the tank bottom. The sector walls were of three types: solid, perforated sheet with 0.020 in. dia. holes and 23% open, and perforated sheet with 0.078 in. dia. holes and 23% open. The direction of translational excitation was as shown in Figure 1.

LIQUID NATURAL FREQUENCIES

The liquid natural frequencies in a sector tank of vertex angle $2\alpha\pi$ are given (3) by the relation

$$\omega_{mn}^2 = \frac{2a}{d} \xi_{mn} \tanh\left(2 \xi_{mn} \frac{h}{d}\right)$$

where ξ_{mn} are the n th roots of

$$J'_{\bar{m}}(\xi_{mn}) = 0, \quad \bar{m} = \frac{m}{2\alpha}$$

For $\bar{m} = 0$, the roots ξ_{mn} have been given in (4), and for $\bar{m} = 1, 2, 3$, the roots have been given by Bauer (3). For $\bar{m} = 4$ through 8, the roots can be obtained by a Lagrangian inverse interpolation from the tables of (5). A complete tabulation of all of these roots are given here in the accompanying table. The roots $\bar{m} = 0, 4$, and 8 apply to a 45° sector tank where, in the notation of Figure 1, the frequencies corresponding to $\bar{m} = 0$ and 8 occur in all sectors other than $3/7$ (line of symmetry normal to the direction of motion), and the frequencies corresponding to $\bar{m} = 4$ occur in all sectors other than $1/5$ (line of symmetry parallel to the direction of motion).

The experimentally determined frequencies were found to be significantly lower than the theoretical values, as was also the case with the quarter tanks (1). Figure 2 shows the variation of the lowest natural frequency ($\bar{m} = 0$), in terms of the dimensionless parameter $\omega_n^2 d / a$, as a function of excitation amplitude λ_0 / d . The improvement in agreement between the measured and the calculated frequency

is quite marked as the excitation amplitude is decreased, as was suggested in (1). In fact, a brief series of measurements with quarter tanks showed that the measured frequencies are significantly increased by reducing the excitation amplitude, thus bringing the frequency data of (1) into a much more satisfactory state. The interesting conclusions to be drawn from these data are, of course, that nonlinearities in the liquid motion are particularly evidenced during sloshing in compartmented tanks and, perhaps even more importantly, the use of compartmentation to raise liquid natural frequencies may be considerably less effective than anticipated. The latter will be true for excitation amplitudes of any appreciable magnitude unless a substantial amount of damping also is introduced to insure that only small amplitude liquid motions will occur.

Even beyond the preceding considerations, compartmented tank design is further complicated if weight reduction by use of perforated compartment walls is envisioned. As was done previously with the quarter tank (1), an investigation of the effects of wall perforation was made in terms of an equivalent Reynolds number (based on perforation hole size d_w). Again it was found that above some critical value of the equivalent Reynolds number the liquid natural frequency decreased quite rapidly, to a value, in fact, approximately that corresponding to an uncompartmented tank (Figure 3). Because of the limited data that could be obtained readily, the dependence of the critical equivalent

Reynolds number on excitation amplitude, as found for the quarter tank in (1), could not be established with any certainty; however, the general behavior and trends are certainly similar. The scatter of values of the frequency at the lower equivalent Reynolds number, and correspondingly in Figure 3 of (1), is the result, of course, of the nonlinearity of liquid frequency with excitation amplitude discussed previously.

FORCED VIBRATION RESPONSE

This data is presented in Figures 4 and 5 in terms of dimensionless force amplitude and phase as a function of excitation frequency. The first of these compares the force response of both 45° and 90° (quarter) sector tanks having solid walls with that of an uncompartmented tank, for a liquid depth of $h/d = 1.0$. Figure 5 shows the effects of wall perforation on force response by comparison with solid wall data.

LIQUID DAMPING

Force response data similar to that of Figure 5 can be employed to determine the effects of wall perforation on liquid damping. While the data is somewhat meager, the variation of damping coefficient γ_s with equivalent Reynolds number (based on perforation hole size) has the general form shown in Figure 6. As anticipated, the liquid damping is significantly increased above the critical value of equivalent Reynolds number as a consequence of the increased liquid interchange between sectors. Although not reported in (1), similar data has also been obtained for quarter tanks.

DISCUSSION

The results of the present study have revealed, more forceably than ever, the complications and numerous interactions involved with liquid sloshing in compartmented tanks. It is seen that the liquid natural frequencies are strongly dependent upon the magnitude of the excitation amplitude, approaching the theoretical values only when the excitation is infinitesimal. Perforation of sector walls is a complicating factor because of the involved relationships between the liquid flow characteristics (described in terms of an equivalent Reynolds number), excitation amplitude, and liquid damping. The designer contemplating the use of compartmentation as a means of avoiding resonances with other system components must therefore be made fully aware of these other complications and interactions.

ACKNOWLEDGEMENT

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TABLE OF VALUES OF ξ_{mn}

ξ_{mn} are nth roots of $-J'_m(\xi_{mn}) = 0$

ξ_{mn}	$\bar{m} = 0$	$\bar{m} = 1$	$\bar{m} = 2$	$\bar{m} = 3$	$\bar{m} = 4$	$\bar{m} = 5$	$\bar{m} = 6$	$\bar{m} = 7$	$\bar{m} = 8$
n = 1	3.8317060	1.841, 19	3.054, 18	4.201, 19	5.317, 55	6.415, 616	7.501, 27	8.577, 84	9.647, 42
n = 2	7.0155867	5.331, 44	6.706, 14	8.015, 24	9.282, 40	10.519, 89	11.734, 97	12.932, 42	14.115, 53
n = 3	10.1734681	8.536, 31	9.969, 42	11.345, 98	12.681, 88	13.987, 17	15.268, 14	16.529, 40	17.773, 98
n = 4	13.3236916	11.706, 00	13.170, 33	14.585, 83	15.964, 06	17.312, 86	18.637, 49	19.941, 90	21.229, 10
n = 5	16.4706301	14.863, 6	16.347, 58	17.788, 73	19.196, 02	20.575, 48	21.931, 76	23.268, 01	24.587, 18

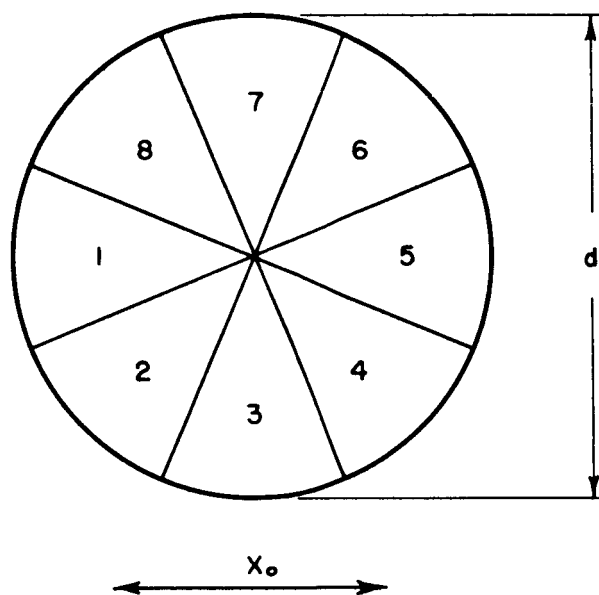


FIGURE 1. TANK CONFIGURATION

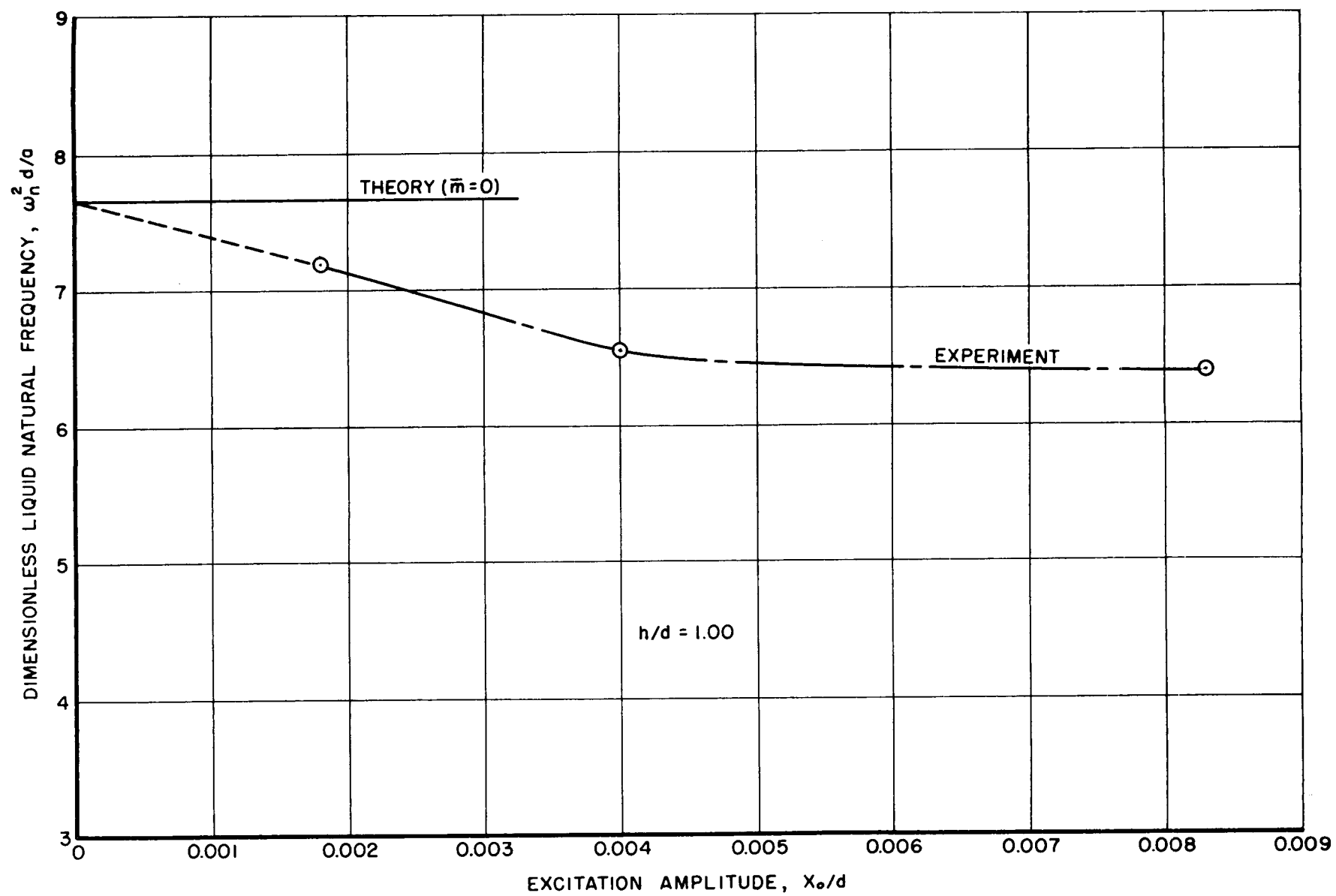


FIGURE 2. EFFECT OF EXCITATION AMPLITUDE ON LOWEST LIQUID NATURAL FREQUENCY

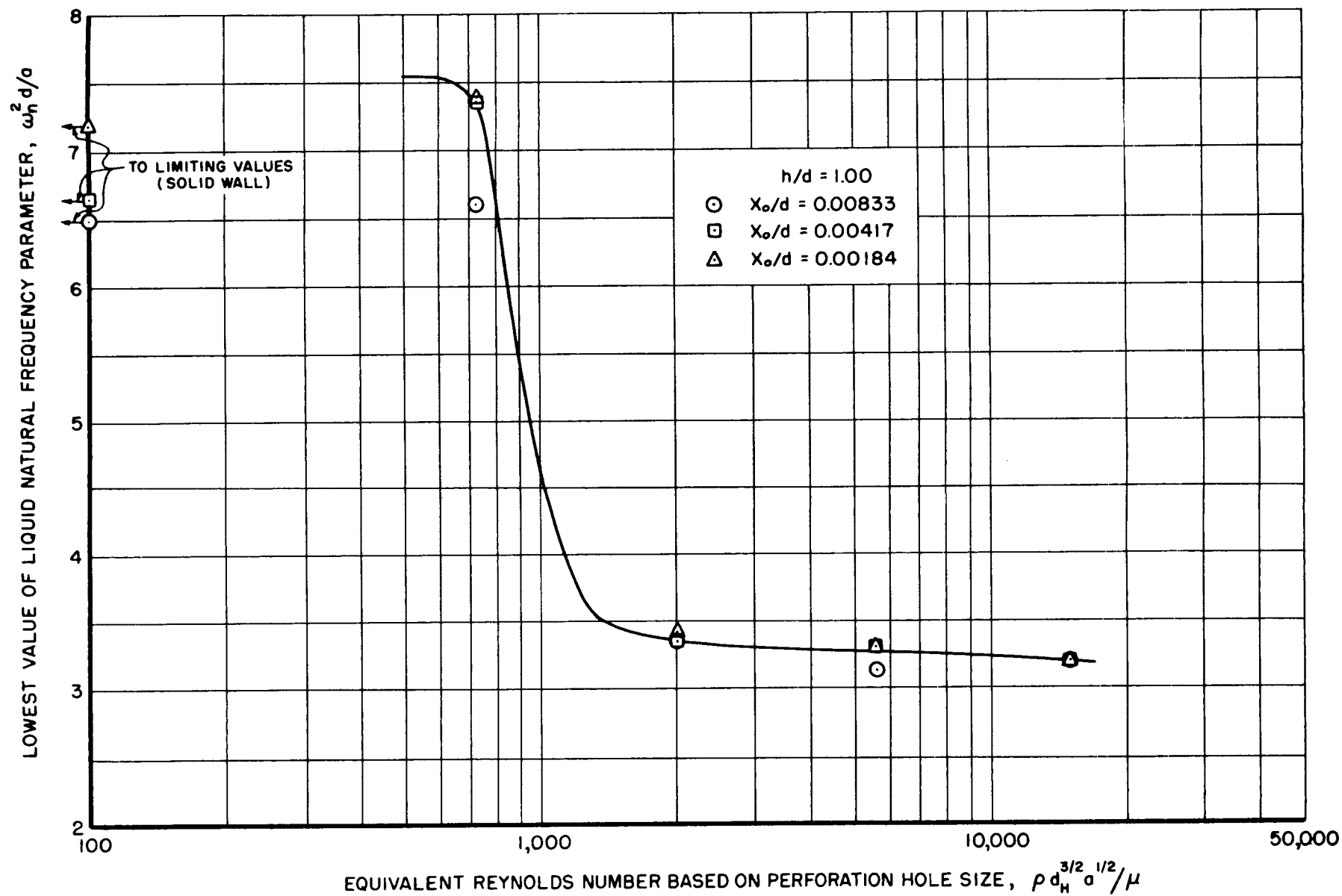


FIGURE 3. VARIATION IN LOWEST LIQUID NATURAL FREQUENCY
FOR A 45° SECTOR TANK WITH PERFORATED WALLS

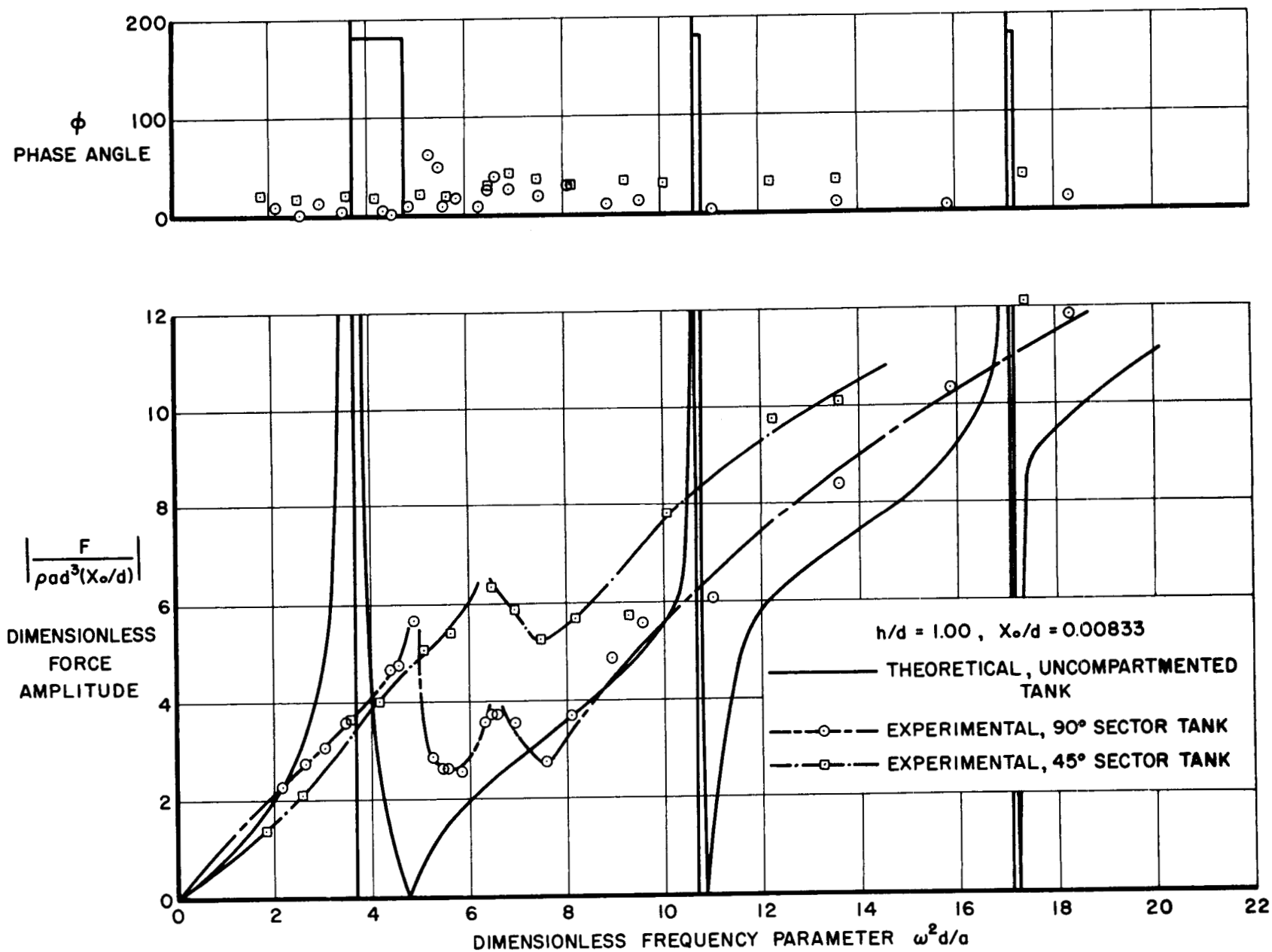


FIGURE 4. TOTAL FORCE RESPONSE FOR 45° AND 90° SECTOR TANKS WITH SOLID WALLS

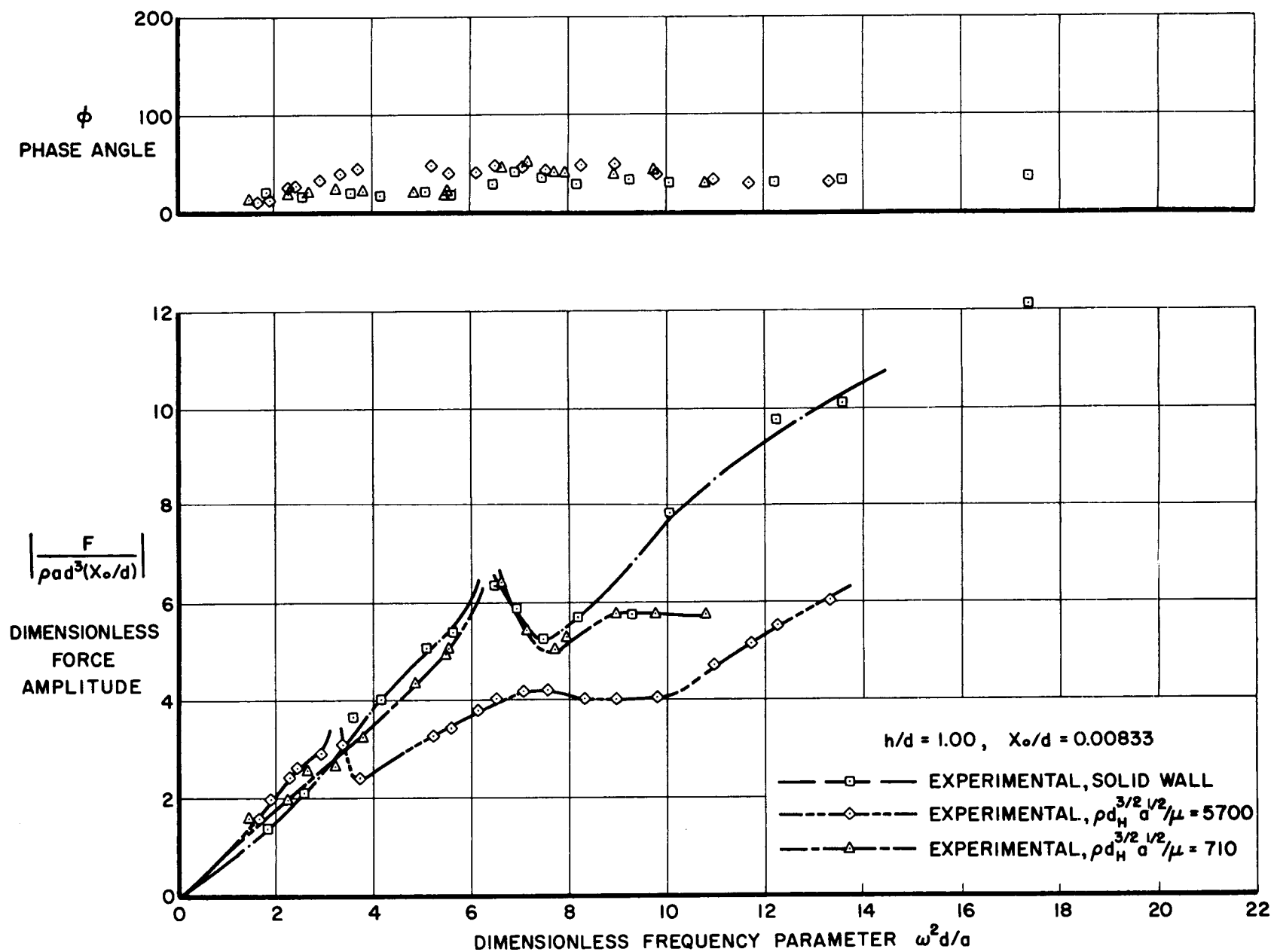


FIGURE 5. TOTAL FORCE RESPONSE FOR 45° SECTOR TANKS WITH PERFORATED WALLS

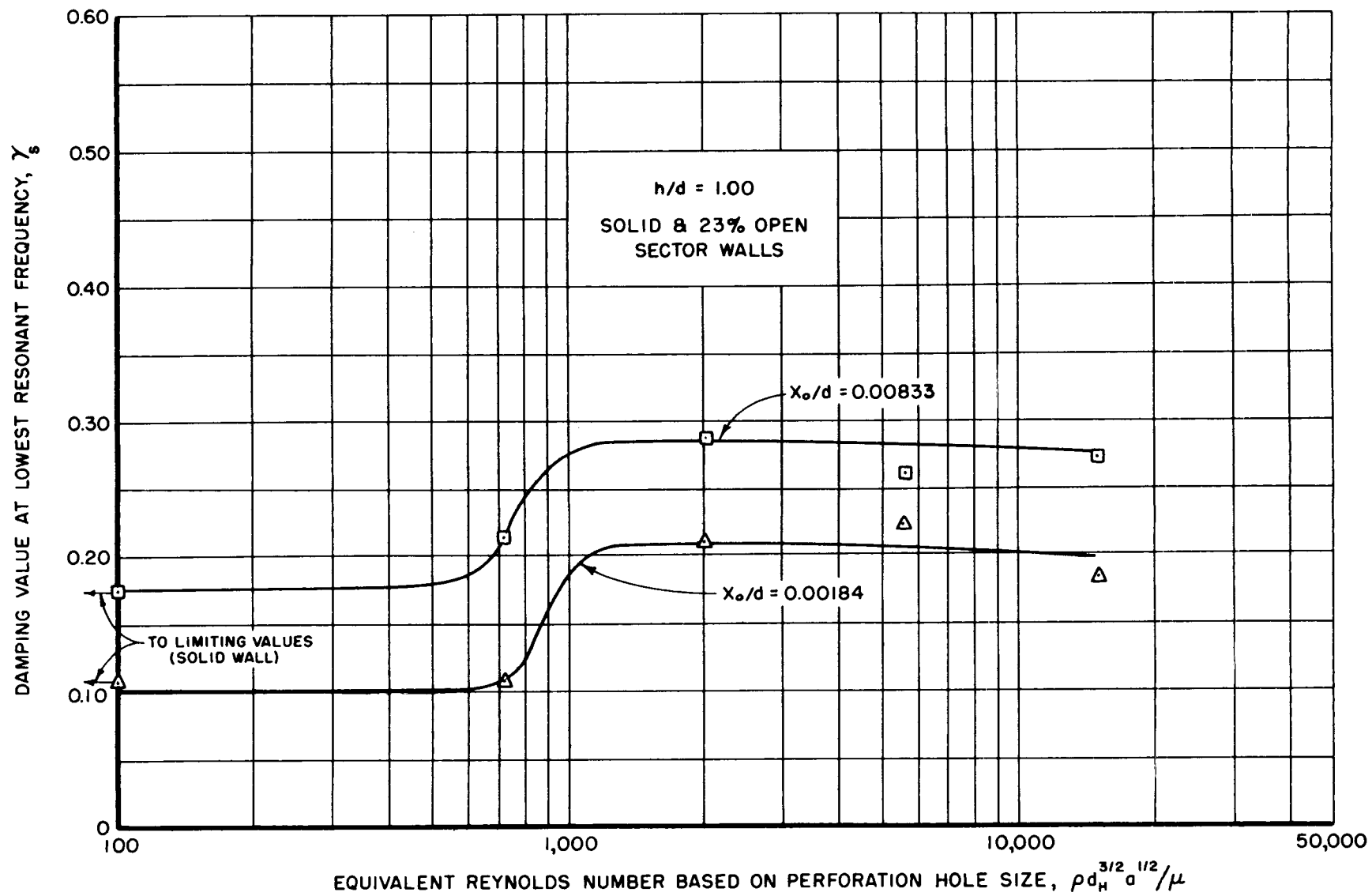


FIGURE 6. LIQUID DAMPING FOR 45° SECTOR TANKS WITH PERFORATED WALLS